



Canadian Mathematics Competition

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Canadian Computing Competition

for the  Awards

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Problem J1

Dressing Up

It is important to keep our computers safe and clean. Some people feel that computers should be well-dressed, also. For this question, you will write a program to print out a bow tie on the computer screen.

Your program should take as input the height H of the bow tie, where H is an odd positive integer greater than or equal to 5. A bow tie with H rows (and $2H$ columns) should then be printed using the pattern shown below. You may assume that all input data will be valid.

For this question you should read the input from the keyboard and print the output to the screen.

Sample Session: (*User input is in italics.*)

Enter height:

5

```
*           *
***       ***
*****
***       ***
*           *
```

Enter height:

7

```
*                 *
***             ***
*****         *****
*****
*****         *****
***             ***
*                 *
```

Problem J2

Mod Inverse

In many cryptographic applications the Modular Inverse is a key point. This question involves finding the modular inverse of a number.

Given $0 < x < m$, where x and m are integers, the modular inverse of x is the unique integer n ,

$0 < n < m$, such that the remainder upon dividing $x \times n$ by m is 1.

For example, $4 \times 13 = 52 = 17 \times 3 + 1$, so the remainder when 52 is divided by 17 is 1, and thus 13 is the inverse of 4 modulo 17.

You are to write a program which accepts as input the two integers x and m , and outputs either the modular inverse n , or the statement "No such integer exists." if there is no such integer n .

Input is from the keyboard, and you may assume that $m \leq 100$.

Output is to the screen.

Sample session: (*user input in italics*)

Enter x:

4

Enter m:

17

13

Enter x:

6

Enter m:

10

No such integer exists.

Problem J3S1

Keeping Score

In a card game, each player's hand is made up of 13 cards. Each hand has a total point value determined by the number of cards that have a point value. The cards which are worth points are the Ace (4 points), King (3 points), Queen (2 points) and Jack (1 point). The other cards (2, 3, 4, 5, 6, 7, 8, 9, 10) have no point value. There are four of each type of card, one in each of the four suits. The suits are called clubs (C), diamonds (D), hearts (H), and spades (S). As well, points are assigned for each suit which has a void (3 points), a singleton (2 points), or a doubleton (1 point). A void in a suit means that there are no cards of that suit (e.g. a hand with no spades). A singleton in a suit means that there is only one card in that suit (e.g. a hand with only one diamond). A doubleton in a suit means that there are only two cards in that suit.

Write a program to read a set of thirteen cards in the form of a string, then evaluate the number of points in the hand. The suits will appear in increasing alphabetical order. Within each suit there will be no duplicate cards.

The output is to be the hand and the point value shown in a table form as below. Your output should list the cards in the same order as the input. Note that 10 is represented by the character T in both the input and the output. Input is from the keyboard, output to the screen.

Sample session (*user input in italics*)

Enter cards:

C258TJKD69QAHSTJA

Cards Dealt	Points
Clubs 2 5 8 T J K	4
Diamonds 6 9 Q A	6
Hearts	3
Spades T J A	5
Total	18

Enter cards:

CAD578KAHAS47TQKA

Cards Dealt	Points
Clubs A	6
Diamonds 5 7 8 K A	7
Hearts A	6
Spades 4 7 T Q K A	9
Total	28

Problem J4S2

Spirals

A spiral of numbers can start and end with any positive integers less than 100. Write a program which will accept two positive integers x and y as input, and output a list of numbers from x to y inclusive, shown in a spiral. You may assume that the end value is greater than or equal to the start value.

A spiral starts with the first number in the centre. The next number appears immediately below the first number. The spiral continues with the numbers increasing in a counter-clockwise direction until the last number is printed.

Read the input from the keyboard and display the output on the screen.

Sample session: (*User input in italics*)

Start value:

10

End value:

27

```

           27  26
16  15  14  25
17  10  13  24
18  11  12  23
19  20  21  22
```

Input starting value:

7

Input end value:

12

```
12  11
 7  10
 8   9
```

Problem J5S3

Strategic Bombing

The Enemy relies heavily on the transportation of supplies and personnel between the specific points A and B. Points A and B, as well as other points C, D, E, etc. are linked by a network of roads. Your mission, should you accept it, is to identify a single road that may be bombed in order to cut off all traffic between A and B.

In the input, each point is identified by a single upper-case letter (there is a maximum of 26). Each line of input identifies a pair of points connected by a road. The end of input is indicated by a line containing "***". All roads are two-way, that is, road AC is the same as road CA. There is at most one road between any pair of points.

Your output should identify all roads such that bombing any one of them would halt all traffic between A and B. Your output should list the roads, one per line, followed by a line stating that "There are n disconnecting roads.", where n is the number of such roads. If there is no such road, output "There are 0 disconnecting roads."

Input is from a file, and output is to a file as specified below.

Sample Input (Input file : bomb.in)

```
AC
AD
AE
CE
CF
ED
GF
BG
HB
GH
**
```

Output for Sample Input (Output file : bomb.out)

```
CF
GF
There are 2 disconnecting roads.
```

Problem S4

Cookies

Making chocolate chip cookies involves mixing flour, salt, oil, baking soda and chocolate chips to form dough which is rolled into a plane. Circles are cut from the plane, placed on a cookie sheet, and baked in an oven for about twenty minutes. When the cookies are done, they are removed from the oven and allowed to cool before being eaten.

We are concerned here with the process of cutting a single round cookie that contains all the chocolate chips. Once the dough has been rolled, each chip is visible in the planar dough, so we need simply to find a cookie cutter big enough to circle all the chips. What is the diameter of the smallest possible round cookie containing all the chips?

Input consists of a positive integer n not greater than 10, followed by n lines of input. Each line give the coordinates of one chocolate chip on the plane. Each coordinate is an integer in the range $[0,1000]$.

Output consists of a single real number, the diameter of the cookie rounded to two decimal places.

Input is from a file, and output is to a file, as specified below.

Sample Input 1 (Input file : cookie.in)

```
4
1 1
1 0
0 1
0 0
```

Output for Sample Input 1 (Output file : cookie.out)

```
1.41
```

Sample Input 2

```
3
1 1
10 0
0 0
```

Output for Sample Input 2

```
10.00
```

Sample Solution

```
#include <stdio.h>
#include <math.h>

double x[10], y[10];
int n;

double diameter(double X, double Y){
    int i;
    double r,max = 0;
    for (i=0;i<n;i++) if ((r=hypot(x[i]-X, y[i]-Y)) > max)
max = r;
    return max*2;
}

main(){
    int i;
    double X,Y,delta;
    scanf("%d",&n);
    for (i=0;i<n;i++) scanf("%lf%lf",&x[i],&y[i]);
    X = Y = 0;
    for (delta=2000;delta > 1e-6;delta *= .9) {
        if (diameter(X+delta,Y) < diameter(X,Y)) X += delta;
        if (diameter(X-delta,Y) < diameter(X,Y)) X -= delta;
        if (diameter(X,Y+delta) < diameter(X,Y)) Y += delta;
        if (diameter(X,Y-delta) < diameter(X,Y)) Y -= delta;
    }
    printf("%lg %lg %0.2lf\n",X,Y,diameter(X,Y));
}
```


Problem S5

Post's Correspondence Problem

Let A and B be two sequences of non-empty strings:

$$A = (a_1, a_2, \dots, a_n), B = (b_1, b_2, \dots, b_n).$$

Let m be a positive integer. Does there exist a sequence of integers

$$i_1, i_2, \dots, i_k \text{ such that } m > k > 0 \text{ and } a_{i_1} a_{i_2} \dots a_{i_k} = b_{i_1} b_{i_2} \dots b_{i_k}$$

For example, if $A = (a, abaaa, ab)$ and $B = (aaa, ab, b)$, then the required sequence of integers is $(2,1,1,3)$ giving $abaaaaaab = abaaaaaab$.

The first two lines of input will contain m and n respectively, and $m \neq n \leq 40$. The next $2n$ lines contain in order the elements of A followed by the elements of B. Each string is at most 20 characters.

If a solution exists, print k on a line by itself, followed by the integer sequence in order, one element per line. Otherwise, print a single line containing "No solution."

Input is from a file, and output is to a file, as specified below.

Sample Input 1 (Input file : post.in)

7
3
a
abaaa
ab
aaa
ab
b

Output for Sample Input 1
(Output file : post.out)

4
2
1
1
3

Sample Input 2

10
3
abc
def
ghi
bcd
efg
hia

Output for Sample Input 2

No solution.